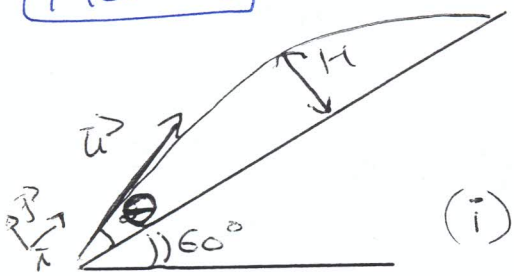
1985 Hons



$g\cos 60\vec{j}$

$g\sin 60\vec{i}$

$\vec{u} = u\cos\theta\vec{i} + u\sin\theta\vec{j}$

$\vec{g} = -g\sin 60\vec{i} - g\cos 60\vec{j}$

$\vec{g} = -\frac{\sqrt{3}}{2}g\vec{i} - \frac{g}{2}\vec{j}$

(i) $\vec{v}(t) = (u\cos\theta - g\frac{\sqrt{3}}{2}t)\vec{i} + (u\sin\theta - \frac{g}{2}t)\vec{j}$

$\vec{r}(t) = (u\cos\theta t - \frac{g\sqrt{3}}{4}t^2)\vec{i} + (u\sin\theta t - \frac{g}{4}t^2)\vec{j}$

(ii) Finding H: Find the time to reach the max height by either finding the time in flight and halving it or noting at max. height

$(\vec{v})_{\vec{j}} = 0 \Rightarrow u\sin\theta - \frac{g}{2}t = 0 \Rightarrow t^* = \frac{2u\sin\theta}{g}$

$\Rightarrow H = (\vec{r}(t^*))_{\vec{j}} = u\sin\theta\left(\frac{2u\sin\theta}{g}\right) - \frac{g}{4}\left(\frac{2u\sin\theta}{g}\right)^2$

$= \frac{2u^2\sin^2\theta}{g} - \frac{u^2\sin^2\theta}{g}$

$H = \frac{u^2\sin^2\theta}{g}$

(iii) $\frac{1}{3}H(\vec{r})_{\vec{j}} = H\sin^2\theta \Rightarrow (\vec{r})_{\vec{j}} = \frac{u^2\sin^4\theta}{g}$

At the times t $\Rightarrow u\sin\theta t - \frac{g}{4}t^2 = \frac{u^2\sin^4\theta}{g}$

This is a quadratic equation in t $\Rightarrow \frac{g}{4}t^2 - u\sin\theta t + \frac{u^2\sin^4\theta}{g} = 0$

$t = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \Rightarrow t = \frac{u\sin\theta \pm \sqrt{(u\sin\theta)^2 - 4\left(\frac{g}{4}\right)\frac{u^2\sin^4\theta}{g}}}{2\left(\frac{g}{4}\right)}$

$\Rightarrow t = \frac{u\sin\theta \pm \sqrt{u^2\sin^2\theta - u^2\sin^4\theta}}{\frac{g}{2}}$

$\Rightarrow t = \frac{u\sin\theta \pm u\sin\theta\sqrt{1-\sin^2\theta}}{\frac{g}{2}}$

$\Rightarrow t = \frac{2}{g}\left[u\sin\theta \pm u\sin\theta\sqrt{\cos^2\theta}\right]$

times at which particle $\Rightarrow t = \frac{2}{g}\left[u\sin\theta \pm u\sin\theta\cos\theta\right]$
is at height $H\sin^2\theta$

$\Rightarrow$ Time above $= \frac{2}{g}\left[u\sin\theta + u\sin\theta\cos\theta\right] - \frac{2}{g}\left[u\sin\theta - u\sin\theta\cos\theta\right]$

$= \frac{2}{g}u\sin\theta\cos\theta + \frac{2u\sin\theta\cos\theta}{g}$

$= \frac{u\sin 2\theta}{g} + \frac{u\sin 2\theta}{g}$

Time above $H\sin^2\theta = \frac{2u\sin 2\theta}{g}$